Motivation

- It is often a tedious job for voters to rank all the candidates.
- Voters may not have all the information to rank all the candidates.
- Want to determine winner from a set of partial votes.
- However, the classical voting rules take complete rankings as input.
- Need to complete the partial votes to determine rankings.
- Winner depends on how we complete.

Preliminaries: Voting

- \( V \) - a set of \( n \) voters.
- \( C \) - a set of \( m \) candidates.
- Complete vote - a complete order over \( C \).
- \( L(C) \) - set of complete orders over \( C \).
- Partial vote - a partial order over \( C \).
- \( V \) - a set of complete votes.
- \( P \) - a set of complete votes.
- \( \langle C, V \rangle \) - a class of scoring rules that includes Borda, Maximin, Copeland, Ranked pairs, and Bucklin.
- Possible Winner: Winner is the candidate with maximum score.
- Bucklin rule: Candidate ranked at \( r \)th position gets score \( \alpha \( r \) \). Winner is the candidate with highest score.
- Maximin rule: Winner is the candidate with minimum margin of victory in its worst pairwise election.
- Copeland rule: Winner is the candidate with maximum pairwise wins.
- Bucklin rule: Winner is the candidate getting majority within minimum number of top positions.

Problem Definition

Possible Winner

Given a voting rule \( r \), a set of partial votes \( P \), and a candidate \( c \), does there exist an completion of the partial votes in \( P \) to linear votes that makes the candidate \( c \) win the election?

Coalitional Manipulation

Given a voting rule \( r \), a set of complete votes \( P \) corresponding to the votes of the non-manipulators, a non-empty set of manipulators \( M \), and a candidate \( c \), does there exist a way to cast the manipulators’ votes such that \( c \) wins the election?

Observation

Coalitional Manipulation is a special case of Possible Winner.

Question: Does there exist kernel of size polynomial in the number of candidates for common voting rules?

Results

Possible Winner does not admit a polynomial kernel for following voting rules unless \( \text{CoNP} \subseteq \text{NP}/\text{Poly} \):
- A class of scoring rules that includes Borda, Maximin, Copeland, Ranked pairs, and Bucklin.

The Coalitional Manipulation problem admits a polynomial kernel for following voting rules:
- A class of scoring rules that includes Borda, Maximin, Copeland, Ranked pairs.

References